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Event based guaranteed cost consensus for distributed multi-agent systems

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Abstract

To investigate the energy consumption involved in an event based control scheme, the problem of event based guaranteed cost consensus for distributed multi-agent systems with general linear time invariant dynamics is considered in this paper. A delay system method is used to transform the multi-agent systems into a special delay system based on a sampled-data event triggering mechanism, which only requires supervision of system states at discrete instants. Sufficient conditions to achieve the consensus with guaranteed cost are presented and expressed as a continuous constrained optimization problem with a linear objective function, linear and bilinear matrix inequalities constraints, involving the co-design of the controller gain matrix and event triggering parameters. An illustrative example is given to show the effectiveness of the proposed approach.

1. Introduction

Multi-agent systems have attracted considerable attention in recent years due to their broad applications in distributed sensor networks, satellite clusters, unmanned aerial vehicle (UAV) formations and robot teams [7,16–18,25,26]. Distributed consensus is a significant problem in multi-agent systems, in which, a group of agents needs to agree on certain quantities of common interest, only sharing information with their neighbors locally [2,29,30]. An important issue in

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the implementation of distributed consensus algorithms comes from the communication and controller actuation schemes. In traditional time-triggered control schemes, the sensor and controller are updated uniformly with a fixed sampling period regardless whether it is necessary or not. Since each agent may be equipped with an embedded microprocessor with limited computing and communication capabilities, event based control (also called event triggered or event driven control) has emerged as an alternative to time triggered control to reduce the number of actuator updates and to facilitate the efficient usage of shared resources. In an event based control scheme, the controllers are updated only when some specific events occur and therefore the frequency of controller updates is reduced [1].

Previous work on event based consensus of multi-agents systems can be found in [4,8,15,24,33] and references therein. To sum up, the mode of event detection can be classified into two groups: (1) continuous event detection; (2) sampled-data event detection. In continuous event detection, event generators have to monitor and check the event triggered conditions constantly and should exclude Zeno behavior [4,8,24]. Obviously, such continuous detection does not sufficiently meet the original requirements to reduce the communication frequency between control components, which may increase the burden of imbedded microprocessors and become an important source of energy consumption. To address the limitations of continuous detection, the concept of sampled-data event detection was proposed in [15], which admits a minimum inter-event time and it is lower bounded by the sampling period; therefore, the Zeno behavior is prohibited automatically. In this study, we will use the sampled-data event detection.

It should be noted that the majority of existing literatures on event based multi-agent consensus problems focus on the case where agents are governed by first-order or second-order dynamics [4,8,15,24], in which, since the control law is predetermined, there is no necessity to design the control gain matrix. In this study, the event based consensus of multi-agent systems with general linear time invariant (LTI) dynamics is considered. This framework not only admits the wide applicability of multi-agents but also allows us to design the controller appropriately.

In general, a stabilization problem in control design can be formulated as a feasibility problem by Lyapunov stability theory from the perspective of optimization. In some cases, for example, the robustness against uncertainty [11,13,20,27,28], to guarantee some level of performance, the concept of guaranteed cost control was firstly introduced in [3], which will yield a standard continuous constrained optimization problem. Taking into account the energy consumption, the event based guaranteed cost consensus is studied, with an objective function involving energy consumption added. To the best of our knowledge, this work that emphasizes the necessity of linking the event based control with guaranteed cost performance from the perspective of energy saving is the first time in multi-agent systems.

The main contribution and novelty of this paper can be summarized as follows: (1) the concept of guaranteed cost is introduced to incorporate with event based control to further investigate the energy consumption; (2) a novel event based consensus approach is applied to general LTI multi-agent systems; (3) the controller gain matrix and the triggering parameters with guaranteed cost performance are co-designed to achieve the consensus for distributed multi-agent systems; (4) a BMI (bilinear matrix inequality) based approach is used to reduce the conservativeness of controller design.

Notations: Throughout this paper, the symmetric terms in a symmetric matrix are denoted by $\begin{pmatrix} X & Y \\ * & Z \end{pmatrix} = \begin{pmatrix} X & Y \\ Y^T & Z \end{pmatrix}$; $\mathbf{1}$ and $\mathbf{11}$, denote a vector and a matrix with all ones, respectively; $P>0$ $(P\geq 0)$ means that P is a real symmetric positive definite (semi-definite) matrix; $\lambda_{\max}(P)$ denotes the maximum eigenvalue of matrix P.

2. Problem formulation

Before starting this section, we will recall some fundamentals which will be used in the sequel.

2.1. Algebraic graph theory

Let $\mathcal{G}=\{\mathcal{V},\mathcal{E}\}$ be a graph with a set of vertices (nodes) $\mathcal{V}=\{1,2,\dots,N\}$ and edges $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$. If there exists an edge (i,j) between two vertices i and j, then i and j are called adjacent, i.e., $\mathcal{E}=\{(i,j)\in\mathcal{V}\times\mathcal{V}: i,j \text{ adjacent}\}$. A graph $\mathcal{G}$ is called undirected if $(i,j)\in\mathcal{E}\leftrightarrow(j,i)\in\mathcal{E}$. A path is a sequence of distinct vertices such that each pair of consecutive vertices is adjacent. If there exists a path from i to j, then i and j are called connected. If all pairs of vertices in a graph $\mathcal{G}$ are connected, then the graph $\mathcal{G}$ is called connected. The adjacency matrix $\mathcal{A}=\{a_{ij}\}^{N\times N}$ is defined by $a_{ij}=1$ if i and j is adjacent and $a_{ij}=0$ otherwise. The degree matrix $\mathcal{D}=\text{diag}\{d_1,d_2,\dots,d_N\}$ is a diagonal matrix with the ith element $d_i=\sum_{j\in N_i}a_{ij}$, here, $N_i=\{j\in\mathcal{V}:(i,j)\in\mathcal{E}\}$ is the neighbor set of agent i. Then the Laplacian matrix L of $\mathcal{G}$ is defined by $L=\mathcal{D}-\mathcal{A}$. For undirected graph, L is symmetric, positive semi-definite, and its row sums are zero, i.e., $L=L^T\geq 0$ and $L\mathbf{1}=\mathbf{0}$.

2.2. Problem statement

Consider the multi-agent systems consisting of a group of N identical agents with a communication graph $\mathcal{G}$. The dynamics of each agent is described by

$$\dot{x}_i(t)=Ax_i(t)+Bu_i(t) \tag{1}$$

where $x_i\in\mathbb{R}^n$ and $u_i\in\mathbb{R}^m$ are the state and the control input of the ith agent, respectively; $A\in\mathbb{R}^{n\times n}$, $B\in\mathbb{R}^{n\times m}$ are constant matrices.

The following distributed consensus protocol is considered

$$u_i(t)=-K\sum_{j\in N_i}(x_i(t)-x_j(t)), \tag{2}$$

where $K\in\mathbb{R}^{m\times n}$ is a control gain matrix to be determined later.

In this paper, an event-triggering mechanism is considered, as illustrated in Fig. 1. For the ith agent, the sensors sample the data at instants kh $(k=0,1,2,\dots)$, here, h is the sampling period. The sampled states $x_i(kh)$ $(i=1,2,\dots,N)$ are transmitted to the event generator and then are only released at their event instants t_k^ih. An event for agent i is triggered as soon as the following

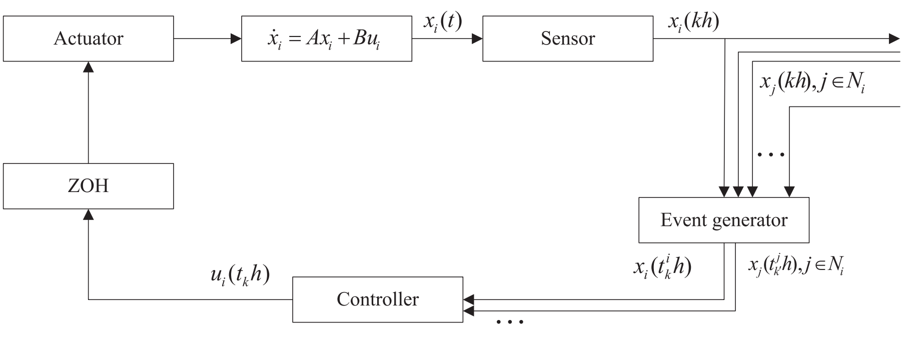


Fig. 1. Structure of the event based multi-agent system.

condition is violated:

$$\left[\sum_{j\in N_i}(\epsilon_i(t)-\epsilon_j(t))\right]^T \Phi\left[\sum_{j\in N_i}(\epsilon_i(t)-\epsilon_j(t))\right]$$
$$\le\rho\left[\sum_{j\in N_i}(x_i(t_k^ih+lh)-x_j(t_{k'}^jh+lh))\right]^T\Phi\left[\sum_{j\in N_i}(x_i(t_k^ih+lh)-x_j(t_{k'}^jh+lh))\right], \quad (3)$$

where $\Phi\in\mathbb{R}^{n\times n}$ is a symmetric positive definite matrix; $l=1,2,\ldots$ is an integer; $\rho\in[0,1)$ is the threshold constant; $t_{k'}^jh=\max\{t|t\in\{t_k^jh,k=0,1,\ldots\},t\le t_k^ih\}$, and $\epsilon_i(t)$ is the measurement error for agent i defined as

$$\epsilon_i(t)=x_i(t_k^ih)-x_i(t_k^ih+lh),\quad t\in[t_k^ih,t_{k+1}^ih), \quad (4)$$

where t_k^ih and t_{k+1}^ih are the current and next event instants, respectively.

Remark 2.1. As shown by the event-triggered condition, the event generator only needs to collect information from its own and its neighbors. The proposed event-triggered conditions have established the relationship between the measure error and the currently sampled state of the ith agent and those of its neighbors. It is not difficult to find that the specific event-triggered condition is designed to adapt to the structure of the distributed consensus protocol (2).

Remark 2.2. The sampled-data event-triggering mechanism only requires detection of system state at discrete instants $(kh, k=0,1,2,\ldots)$. The set of release instants $\{t_0^i,t_1^i,t_2^i,\ldots\}$ $(i=1,2,\ldots,N)$ should be a proper subset of $\{0,1,2,\ldots\}$, which is depending on the triggering parameters Φ and ρ as well as the dynamics of multi-agent systems. If $\rho=0$, the event based scheme reduces to a time triggered scheme.

Let the latest broadcast state of agent i be defined as

$$\hat{x}_i(t)=x_i(t_k^ih),\quad t\in[t_k^ih,t_{k+1}^ih), \quad (5)$$

which converts the discrete signal $x_i(t_k^ih)$ into the continuous signal $\hat{x}_i(t)$ by holding it constant until the next event instant. Under the notation defined, for $[t_k^ih,t_{k+1}^ih)$, the closed-loop system for agent i can be obtained by

$$\dot{x}_i(t)=Ax_i(t)-BK\sum_{j\in N_i}\left(\hat{x}_i(t)-\hat{x}_j(t)\right)$$
$$=Ax_i(t)-BK\sum_{j\in N_i}\left(x_i(t_k^ih)-x_j(t_{k'}^jh)\right). \quad (6)$$

Then, a delay system method [31] is used to transform the multi-agent systems (1) based on sampled-data event-triggering mechanism to a special delay system.

Let $\mathcal{T}_0^i=[t_k^ih,t_k^ih+h)$, $\mathcal{T}_l^i=[t_k^ih+lh,t_k^ih+(l+1)h)$, $l=1,2,\ldots,l_m^i$. Here, l_m^i is the largest integer that satisfies Eq. (3). Then, $t_{k+1}^ih=(l_m^i+1)h$, and we have

$$\bigcup_{l=0}^{l=l_m^i}\mathcal{T}_l^i=[t_k^ih,t_{k+1}^ih). \quad (7)$$

A time-varying delay is defined as follows:

$$\tau_i(t)=\begin{cases} t-t_k^i h, & t\in\mathcal{T}_0^i \\ t-t_k^i h-lh, & t\in\mathcal{T}_l^i,\ \ l=1,2,\ldots,l_m^i \end{cases}$$

It is easy to find that $0\le\tau_i(t)\le h$.

Remark 2.3. The time-varying delay $\tau_i(t)$ can be regarded as the same for different agents. To explain, consider the jth ($j\neq i$) agent.

Let $\mathcal{T}_0^j=[t_{k'}^j h, t_{k'}^j h+h)$, $\mathcal{T}_{l'}^j=[t_{k'}^j h+l'h, t_{k'}^j h+(l'+1)h), l'=1,2,\ldots,l_m^j$ (l_m^j has the similar meaning as l_m^i), the measurement error for the jth agent is defined by

$$\epsilon_j(t)=x_j(t_{k'}^j h)-x_j(t_{k'}^j h+l'h),\quad t\in\mathcal{T}_0^j\cup\mathcal{T}_{l'}^j$$

and the corresponding time-varying delay is defined as follows:

$$\tau_j(t)=\begin{cases} t-t_{k'}^j h, & t\in\mathcal{T}_0^j, \\ t-t_{k'}^j h-l'h, & t\in\mathcal{T}_{l'}^j,\ \ l'=1,2,\ldots,l_m^j. \end{cases}$$

Although it seems that expressions of $\tau_i(t)$ for the ith agent and $\tau_j(t)$ are different, i.e., $t_k^i h$ and $t_{k'}^j h$ are different, it should be noted that the domains of $\tau_i(t)$ and $\tau_j(t)$, i.e. $\mathcal{T}_0^i$ and $\mathcal{T}_0^j$, $\mathcal{T}_l^i$ and $\mathcal{T}_{l'}^j$ are also different. According to the definitions of domains $\mathcal{T}_0^i$ and $\mathcal{T}_0^j$, $\mathcal{T}_l^i$ and $\mathcal{T}_{l'}^j$, it is not difficult to find that, under any interval $[kh, kh+h)$, $\tau_i(t)$ and $\tau_j(t)$ have the same characteristics. Thus, $\tau_i(t)$ and $\tau_j(t)$ can be considered as the same $\tau(t)$.

Then, for each $i\in\mathcal{V}$ and $t\in[t_k^i h+lh, t_k^i h+lh+h)$, which corresponds to an interval $\mathcal{T}_{l'}^j=[t_{k'}^j h+l'h, t_{k'}^j h+(l'+1)h)$ for $j\in\mathcal{V}$, according to the above definition of $\tau_i(t)$ and $\tau_j(t)$, we have

$$\begin{aligned}\dot{x}_i(t)&=Ax_i(t)-BK\sum_{j\in N_i}\left(x_i(t_k^i h)-x_j(t_{k'}^j h)\right)\\ &=Ax_i(t)-BK\sum_{j\in N_i}\left(x_i(t_k^i h+lh)-x_j(t_{k'}^j h+l'h)\right)\\ &\quad -BK\sum_{j\in N_i}\left[\left(x_i(t_k^i h)-x_i(t_k^i h+lh)\right)-\left(x_j(t_{k'}^j h)-x_j(t_{k'}^j h+l'h)\right)\right]\\ &=Ax_i(t)-BK\sum_{j\in N_i}\left(x_i(t-\tau_i(t))-x_j(t-\tau_j(t))\right)-BK\sum_{j\in N_i}\left(\epsilon_i(t)-\epsilon_j(t)\right)\\ &=Ax_i(t)-BK\sum_{j\in N_i}\left(x_i(t-\tau(t))-x_j(t-\tau(t))\right)-BK\sum_{j\in N_i}\left(\epsilon_i(t)-\epsilon_j(t)\right)\end{aligned}\tag{8}$$

with the initial condition $x_i(t)=e^{At}x_i(0), t\in[-h,0]$.

Moreover, the triggering condition can be reformulated as

$$\begin{aligned}&\left[\sum_{j\in N_i}(\epsilon_i(t)-\epsilon_j(t))\right]^T\Phi\left[\sum_{j\in N_i}(\epsilon_i(t)-\epsilon_j(t))\right]\\ &\quad\le\rho\left[\sum_{j\in N_i}(x_i(t-\tau(t))-x_j(t-\tau(t)))\right]^T\Phi\left[\sum_{j\in N_i}(x_i(t-\tau(t))-x_j(t-\tau(t)))\right].\end{aligned}\tag{9}$$

Let $x(t) = [x_1^T(t), \ldots, x_N^T(t)]^T$, $x(t-\tau(t)) = [x_1^T(t-\tau(t)), \ldots, x_N^T(t-\tau(t))]^T$, $\epsilon(t) = [\epsilon_1^T(t), \ldots, \epsilon_N^T(t)]^T$, by using the Kronecker product for representation, the compact form of the multi-agent system can be expressed as follows:

$$\dot{x}(t) = (I_N \otimes A)x(t) - (L \otimes BK)x(t-\tau(t)) - (L \otimes BK)\epsilon(t). \tag{10}$$

For given $Q^T = Q > 0$ and $R^T = R > 0$ with appropriate dimensions, the quadratic cost functional associated with the multi-agent system is given by

$$J = \sum_{i=1}^{N} \int_0^{\infty} \left[\sum_{j \in N_i} \left(x_i(t) - x_j(t) \right) \right]^T Q \left[\sum_{j \in N_i} \left(x_i(t) - x_j(t) \right) \right] + u_i(t)^T R u_i(t)\, dt. \tag{11}$$

Remark 2.4. The consensus problem of the multi-agent systems with quadratic cost functional is similar to a typical LQR (linear-quadratic regulator) problem. The quadratic cost functional (11) can be regarded as a performance metric on energy consumption during consensus evolution of multi-agent systems.

3. Main results

Existing mode of consensus can be categorized into two classes: consensus without a leader (leaderless consensus) and consensus with a leader (leader–follower consensus) [5,14]. In this study, the leaderless consensus is considered.

We introduce the following definition.

Definition 3.1. The multi-agent system (1) is said to achieve event based guaranteed cost consensus under the distributed consensus protocol (2) and the event-triggering criteria (3), if there exist a gain matrix K, triggering parameters Φ and ρ such that $\lim_{t \to \infty} \| x_i(t) - x_j(t) \| = 0$, $\forall\ i, j \in \mathcal{V}$ and the value of the quadratic cost functional (11) is bounded from above, i.e., $J \le J^*$, and J^* is said to be a guaranteed cost.

Define new variables as

$$\xi_i(t) = x_i(t) - \frac{1}{N} \sum_{j=1}^{N} x_j(t),$$

and $\xi(t) = [\xi_1^T(t), \ldots, \xi_N^T(t)]^T$. Then, it is easy to find that $\xi(t) = 0$ if and only if $x_1(t) = x_2(t) = \cdots = x_N(t)$.

To simplify the operations, some useful symbols are defined in the following.

Let $E_i = [\mathbf{0}, \ldots, \underbrace{I}_{i}, \ldots, \mathbf{0}] \in \mathbb{R}^{n \times Nn}$ be a block matrix and $E = \sum_{i=1}^{N} E_i$, here, I is the ith identity matrix with appropriate dimension.

Let $L \otimes BK$ be partitioned conformably as

$$L \otimes BK = \begin{pmatrix} (L \otimes BK)_1 \\ \cdots \\ (L \otimes BK)_N \end{pmatrix} \in \mathbb{R}^{Nn \times Nn}$$

where $(L \otimes BK)_i \in \mathbb{R}^{n \times Nn}$.

Property 3.1. *Suppose S and T are matrices with appropriate dimensions, it is easy to verify the following properties*:

$$\sum_{i=1}^{N} E_i^T S E_i = I_N \otimes S;$$

$$\sum_{i=1}^{N} E_i^T S E = \sum_{i=1}^{N} E^T S E_i = E^T S E = \mathbf{1}\mathbf{1}^T \otimes S;$$

$$\sum_{i=1}^{N} (L \otimes S)_i = \mathbf{1}^T L \otimes S = \mathbf{0};$$

$$\sum_{i=1}^{N} E_i^T S (L \otimes T)_i = L \otimes ST;$$

$$\sum_{i=1}^{N} (L \otimes T)_i^T S (L \otimes T)_i = (L^T L) \otimes (T^T S T).$$

Lemma 3.1 (*Jensen inequality, Gu et al. [10]*). *For a given symmetric positive definite matrix $Z>0$ and for any differentiable function $f:[a,b]\to\mathbb{R}^n$, the following inequality holds*:

$$\int_a^b \dot{f}^T(s) Z \dot{f}(s)\, ds \geq \frac{1}{b-a}[f(a)-f(b)]^T Z [f(a)-f(b)].$$

Lemma 3.2 (*Reciprocally convex combination, Park et al. [19]*). *Suppose $f_1, f_2, \ldots, f_N : \mathbb{R}^m \mapsto \mathbb{R}$ have positive values in an open subset $\mathcal{S}$ of $\mathbb{R}^m$. Then, the reciprocally convex combination of f_i over $\mathcal{S}$ satisfies*

$$\min_{\{\alpha_i | \alpha_i > 0, \sum_i \alpha_i = 1\}} \sum_i \frac{1}{\alpha_i} f_i(v) = \sum_i f_i(v) + \max_{g_{i,j}(v)} \sum_{i \neq j} g_{i,j}(v)$$

subject to

$$\left\{ g_{i,j} : \mathbb{R}^m \mapsto \mathbb{R},\ g_{j,i}(v) = g_{i,j}(v),\ \begin{bmatrix} f_i(v) & g_{i,j}(v) \\ g_{i,j}(v) & f_j(v) \end{bmatrix} \geq 0 \right\}.$$

Let $\chi(t) = [x^T(t)\ x^T(t-\tau(t))\ x^T(t-h)\ \epsilon^T(t)]^T$ and the corresponding block entry matrices be

$$e_1 = [I\ 0\ 0\ 0]^T, \quad e_2 = [0\ I\ 0\ 0]^T, \quad e_3 = [0\ 0\ I\ 0]^T, \quad e_4 = [0\ 0\ 0\ I]^T. \tag{12}$$

Now, we are ready to present our main result in this paper.

Theorem 3.1. *The multi-agent system (1) is said to achieve event based guaranteed cost consensus under the distributed consensus protocol (2) and the event-triggering criteria (3), if there exist a gain matrix K, triggering parameters Φ, $0 \leq \rho < 1$, real matrices $P = P^T > 0$, $Z_1 = Z_1^T$, $Z_2 = Z_2^T$ and S with appropriate dimensions, such that the following bilinear matrix inequalities (BMIs) are satisfied*:

$$\begin{bmatrix} \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4 & \Pi_5 \\ * & \Pi_6 & \Pi_7 & \mathbf{0} & \Pi_8 \\ * & * & \Pi_9 & \mathbf{0} & \mathbf{0} \\ * & * & * & \Pi_{10} & \Pi_{11} \\ * & * & * & * & \Pi_{12} \end{bmatrix} \leq \mathbf{0}, \tag{13}$$

where

$$\begin{aligned}
\Pi_1 &= \left(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\right) \otimes \left(PA + A^T P + Z_1 - Z_2\right) - \frac{h^2}{N}\mathbf{1}\mathbf{1}^T \otimes A^T Z_2 A + L^2 \otimes Q + L^2 \\
&\quad \otimes \left(K^T R K\right), \\
\Pi_2 &= -L \otimes PBK + \left(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\right) \otimes \left(Z_2 - S^T\right), \\
\Pi_3 &= \left(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\right) \otimes S^T, \\
\Pi_4 &= -L \otimes PBK, \\
\Pi_5 &= hI_N \otimes A^T Z_2, \\
\Pi_6 &= \left(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\right) \otimes \left(-2Z_2 + S + S^T\right) + \rho L^2 \otimes \Phi, \\
\Pi_7 &= \left(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\right) \otimes \left(Z_2 - S^T\right), \\
\Pi_8 &= -hL \otimes K^T B^T Z_2, \\
\Pi_9 &= -\left(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\right) \otimes (Z_1 + Z_2), \\
\Pi_{10} &= -L^2 \otimes \Phi, \\
\Pi_{11} &= -hL \otimes K^T B^T Z_2, \\
\Pi_{12} &= -I_N \otimes Z_2.
\end{aligned}$$

Moreover, the value of the quadratic cost functional (11) satisfies

$$\begin{aligned}
J \le J^* &= x^T(0)\left[\left(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\right) \otimes P\right]x(0) + \int_{-h}^{0} x^T(s)\left[\left(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\right) \otimes Z_1\right]x(s)\,ds \\
&\quad + h\int_{-h}^{0}\int_{\theta}^{0} \dot{x}^T(s)\left[\left(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\right) \otimes Z_2\right]\dot{x}(s)\,ds\,d\theta.
\end{aligned}$$

Proof. Consider the following Lyapunov functional candidate:

$$V(t) = V_1(t) + V_2(t) + V_3(t) \tag{14}$$

where

$$\begin{aligned}
V_1(t) &= \sum_{i=1}^{N} \xi_i^T(t) P \xi_i(t) \\
V_2(t) &= \sum_{i=1}^{N} \int_{t-h}^{t} \xi_i(s)^T Z_1 \xi_i(s)\,ds \\
V_3(t) &= h\sum_{i=1}^{N} \int_{t-h}^{t}\int_{\theta}^{t} \dot{\xi}_i(s)^T Z_2 \dot{\xi}_i(s)\,ds\,d\theta
\end{aligned}$$

The time derivative of $V(t)$ along the trajectory of the multi-agent systems becomes

$$\dot{V}_1(t) = 2\sum_{i=1}^{N} \xi_i^T(t) P \dot{\xi}_i(t)$$

$$
\begin{aligned}
&= 2\sum_{i=1}^{N}\left[x_i(t)-\frac{1}{N}\sum_{j=1}^{N}x_j(t)\right]^T P\left[\dot{x}_i(t)-\frac{1}{N}\sum_{j=1}^{N}\dot{x}_j(t)\right]\\
&= 2\sum_{i=1}^{N}\left[x_i(t)-\frac{1}{N}\sum_{j=1}^{N}x_j(t)\right]^T P\left\{Ax_i(t)-BK\sum_{j\in N_i}(x_i(t-\tau(t))-x_j(t-\tau(t)))\right.\\
&\quad -BK\sum_{j\in N_i}\big(\epsilon_i(t)-\epsilon_j(t)\big)-\frac{1}{N}\sum_{j=1}^{N}\left[Ax_j(t)-BK\sum_{k\in N_j}(x_j(t-\tau(t))\right.\\
&\quad \left.\left.-x_k(t-\tau(t)))-BK\sum_{k\in N_j}(\epsilon_j(t)-\epsilon_k(t))\right]\right\}\\
&= 2\sum_{i=1}^{N}\left[E_i x(t)-\frac{1}{N}Ex(t)\right]^T P\Big\{AE_i x(t)-(L\otimes BK)_i x(t-\tau(t))\\
&\quad -(L\otimes BK)_i\epsilon(t)-\frac{1}{N}AEx(t)\Big\}\\
&= \chi^T(t)\left\{e_1\left[\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes\left(PA+A^TP\right)\right]e_1^T-e_1[L\otimes PBK]e_2^T\right.\\
&\quad \left.-e_2[(L\otimes PBK)^T]e_1^T-e_1[L\otimes PBK]e_4^T+e_4[(L\otimes PBK)^T]e_1^T\right\}\chi(t)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_2(t) &= \sum_{i=1}^{N}\left[\xi_i(t)^T Z_1\xi_i(t)-\xi_i(t-h)^T Z_1\xi_i(t-h)\right]\\
&= \sum_{i=1}^{N}\left\{\left[x_i(t)-\frac{1}{N}\sum_{j=1}^{N}x_j(t)\right]^T Z_1\left[x_i(t)-\frac{1}{N}\sum_{j=1}^{N}x_j(t)\right]\right.\\
&\quad \left.-\left[x_i(t-h)-\frac{1}{N}\sum_{j=1}^{N}x_j(t-h)\right]^T Z_1\left[x_i(t-h)-\frac{1}{N}\sum_{j=1}^{N}x_j(t-h)\right]\right\}\\
&= \sum_{i=1}^{N}\left\{\left[E_i x(t)-\frac{1}{N}Ex(t)\right]^T Z_1\left[E_i x(t)-\frac{1}{N}Ex(t)\right]\right.\\
&\quad \left.-\left[E_i x(t-h)-\frac{1}{N}Ex(t-h)\right]^T Z_1\left[E_i x(t-h)-\frac{1}{N}Ex(t-h)\right]\right\}\\
&= x^T(t)\left[\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_1\right]x(t)-x^T(t-h)\left[\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_1\right]x(t-h)\\
&= \chi^T(t)\left\{e_1\left[\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_1\right]e_1^T-e_3\left[\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_1\right]e_3^T\right\}\chi(t)
\end{aligned}
$$

$$
\begin{aligned}
\dot{V}_3(t) &= \sum_{i=1}^{N}[h^2\dot{\xi}_i(t)^T Z_2\dot{\xi}_i(t)-h\int_{t-h}^{t}\dot{\xi}_i(s)^T Z_2\dot{\xi}_i(s)\,ds]\\
&= \sum_{i=1}^{N}[h^2\dot{\xi}_i(t)^T Z_2\dot{\xi}_i(t)-h\int_{t-h}^{t-\tau(t)}\dot{\xi}_i(s)^T Z_2\dot{\xi}_i(s)\,ds-h\int_{t-\tau(t)}^{t}\dot{\xi}_i(s)^T Z_2\dot{\xi}_i(s)\,ds]
\end{aligned}
$$

By using Lemmas 3.1 and 3.2, we have

$$\begin{aligned}
\dot{V}_3(t) \le & \sum_{i=1}^{N}\left\{h^2\left[\dot{x}_i(t)-\frac{1}{N}\sum_{j=1}^{N}\dot{x}_j(t)\right]^T Z_2\left[\dot{x}_i(t)-\frac{1}{N}\sum_{j=1}^{N}\dot{x}_j(t)\right]\right.\\
& -\frac{h}{h-\tau(t)}[\xi_i(t-\tau(t))-\xi_i(t-h)]^T Z_2[\xi_i(t-\tau(t))-\xi_i(t-h)]\\
& \left.-\frac{h}{\tau(t)}[\xi_i(t)-\xi_i(t-\tau(t))]^T Z_2[\xi_i(t)-\xi_i(t-\tau(t))]\right\}\\
\le & \sum_{i=1}^{N}\left\{h^2\left[Ax_i(t)-BK\sum_{j\in N_i}(x_i(t-\tau(t))-x_j(t-\tau(t)))-BK\sum_{j\in N_i}(\epsilon_i(t)-\epsilon_j(t))\right.\right.\\
& \left.-\frac{1}{N}\sum_{j=1}^{N}\left(Ax_j(t)-BK\sum_{k\in N_j}(x_j(t-\tau(t))-x_k(t-\tau(t)))-BK\sum_{k\in N_j}(\epsilon_j(t)-\epsilon_k(t))\right)\right]^T\cdot\\
& Z_2\left[Ax_i(t)-BK\sum_{j\in N_i}(x_i(t-\tau(t))-x_j(t-\tau(t)))-BK\sum_{j\in N_i}(\epsilon_i(t)-\epsilon_j(t))\right.\\
& \left.-\frac{1}{N}\sum_{j=1}^{N}\left(Ax_j(t)-BK\sum_{k\in N_j}(x_j(t-\tau(t))-x_k(t-\tau(t)))-BK\sum_{k\in N_j}(\epsilon_j(t)-\epsilon_k(t))\right)\right]\\
& \left.-\chi^T(t)\begin{bmatrix} e_2^T-e_3^T\\ e_1^T-e_2^T\end{bmatrix}^T\begin{bmatrix}\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_2 & \left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes S\\ \left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes S^T & \left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_2\end{bmatrix}\begin{bmatrix} e_2^T-e_3^T\\ e_1^T-e_2^T\end{bmatrix}\chi(t)\right\}\\
= & \sum_{i=1}^{N}\left\{h^2\left[AE_ix(t)-(L\otimes BK)_i x(t-\tau(t))-(L\otimes BK)_i\epsilon(t)-\frac{1}{N}AEx(t)\right]^T Z_2\right.\\
& \cdot\left[AE_ix(t)-(L\otimes BK)_i x(t-\tau(t))-(L\otimes BK)_i\epsilon(t)-\frac{1}{N}AEx(t)\right]\\
& \left.-\chi^T(t)\begin{bmatrix} e_2^T-e_3^T\\ e_1^T-e_2^T\end{bmatrix}^T\begin{bmatrix}\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_2 & \left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes S\\ \left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes S^T & \left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_2\end{bmatrix}\begin{bmatrix} e_2^T-e_3^T\\ e_1^T-e_2^T\end{bmatrix}\chi(t)\right\}\\
= & \chi^T(t)\left\{h^2e_1\left[\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes A^TZ_2A\right]e_1^T-h^2e_1\left[L\otimes A^TZ_2BK\right]e_2^T\right.\\
& -h^2e_2(L\otimes A^TZ_2BK)^T\big]e_1^T-h^2e_1\left[L\otimes A^TZ_2BK\right]e_4^T-h^2e_4\left[(L\otimes A^TZ_2BK)^T\right]e_1^T\\
& +h^2e_2\left[L^2\otimes\left(K^TB^TZ_2BK\right)\right]e_4^T+h^2e_4\left[L^2\otimes\left(K^TB^TZ_2BK\right)\right]e_2^T\\
& +h^2e_2\left[L^2\otimes\left(K^TB^TZ_2BK\right)\right]e_2^T+h^2e_4\left[L^2\otimes\left(K^TB^TZ_2BK\right)\right]e_4^T\\
& \left.-\begin{bmatrix} e_2^T-e_3^T\\ e_1^T-e_2^T\end{bmatrix}^T\begin{bmatrix}\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_2 & \left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes S\\ \left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes S^T & \left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_2\end{bmatrix}\begin{bmatrix} e_2^T-e_3^T\\ e_1^T-e_2^T\end{bmatrix}\right\}\chi(t)
\end{aligned}$$

Applying the event-triggering conditions (3), we have

$$\dot{V}(t)\le\dot{V}(t)-\sum_{i=1}^{N}\left[\sum_{j\in N_i}(\epsilon_i(t)-\epsilon_j(t))\right]^T\Phi\left[\sum_{j\in N_i}(\epsilon_i(t)-\epsilon_j(t))\right]$$

$$
\begin{aligned}
&+\sum_{i=1}^{N}\rho\left[\sum_{j\in N_i}(x_i(t-\tau(t))-x_j(t-\tau(t)))\right]^T\Phi\left[\sum_{j\in N_i}(x_i(t-\tau(t))-x_j(t-\tau(t)))\right]\\
&=\dot{V}(t)-\chi^T(t)e_4[L^2\otimes\Phi]e_4^T\chi(t)+\rho\chi^T(t)e_2[L^2\otimes\Phi]e_2^T\chi(t)\\
&=\dot{\tilde{V}}(t)
\end{aligned}
$$

Now, consider the quadratic cost functional (11). Since

$$
\sum_{i=1}^{N}\left[\sum_{j\in N_i}(x_i(t)-x_j(t))\right]^T Q\left[\sum_{j\in N_i}(x_i(t)-x_j(t))\right]=\chi^T(t)e_1(L^2\otimes Q)e_1^T\chi(t),
$$

$$
\sum_{i=1}^{N}u_i(t)^TRu_i(t)=\chi^T(t)e_1(L^2\otimes K^TRK)e_1^T\chi(t),
$$

we have

$$
\dot{V}(t)\le\dot{\tilde{V}}(t)\le\dot{\tilde{V}}(t)+\chi^T(t)[e_1(L^2\otimes Q)e_1^T+e_1(L^2\otimes K^TRK)e_1^T]\chi(t)=\chi^T(t)\Omega\chi(t),
$$

where

$$
\begin{aligned}
\Omega=\;&e_1\left[\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes\left(PA+A^TP\right)\right]e_1^T-e_1[L\otimes PBK]e_2^T-e_2\left[(L\otimes PBK)^T\right]e_1^T\\
&-e_1[L\otimes PBK]e_4^T-e_4\left[(L\otimes PBK)^T\right]e_1^T\\
&+e_1\left[\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_1\right]e_1^T-e_3\left[\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_1\right]e_3^T\\
&+h^2e_1\left[\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes A^TZ_2A\right]e_1^T-h^2e_1\left[L\otimes A^TZ_2BK\right]e_2^T\\
&-h^2e_2\left[(L\otimes A^TZ_2BK)^T\right]e_1^T-h^2e_1\left[L\otimes A^TZ_2BK\right]e_4^T-h^2e_4\left[(L\otimes A^TZ_2BK)^T\right]e_1^T\\
&+h^2e_2\left[L^2\otimes\left(K^TB^TZ_2BK\right)\right]e_4^T+h^2e_4\left[L^2\otimes\left(K^TB^TZ_2BK\right)\right]e_2^T\\
&+h^2e_2\left[L^2\otimes\left(K^TB^TZ_2BK\right)\right]e_2^T+h^2e_4\left[L^2\otimes\left(K^TB^TZ_2BK\right)\right]e_4^T\\
&-\begin{bmatrix}e_2^T-e_3^T\\ e_1^T-e_2^T\end{bmatrix}^T\begin{bmatrix}\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_2 & \left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes S\\ \left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes S^T & \left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_2\end{bmatrix}\begin{bmatrix}e_2^T-e_3^T\\ e_1^T-e_2^T\end{bmatrix}\\
&-e_4(L^2\otimes\Phi)e_4^T+\rho e_2(L^2\otimes\Phi)e_2^T\\
&+e_1(L^2\otimes Q)e_1^T+e_1(L^2\otimes K^TRK)e_1^T
\end{aligned}
$$

It can be seen that if $\Omega\le 0$, then $\dot{V}(t)\le 0$, which implies $\lim_{t\to\infty}\xi_i(t)=0, i\in\mathcal{V}$, i.e., $\lim_{t\to\infty}\|x_i(t)-x_j(t)\|=0,\ \forall\ i,j\in\mathcal{V}$.

To continue, we can see that $\Omega\le 0$ is equivalent to the following matrix form:

$$
\begin{bmatrix}
\Pi_1+\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_2 & -L\otimes PBK & \mathbf{0} & -L\otimes PBK\\
* & \rho(L^2\otimes\Phi) & \mathbf{0} & \mathbf{0}\\
* & * & -\left(I_N-\frac{1}{N}\mathbf{1}\mathbf{1}^T\right)\otimes Z_1 & \mathbf{0}\\
* & * & * & -L^2\otimes\Phi
\end{bmatrix}
$$

$$
\begin{aligned}
&+\begin{bmatrix} hI_N \otimes A^T \\ -hL \otimes K^T B^T \\ \mathbf{0} \\ -hL \otimes K^T B^T \end{bmatrix} (I_N \otimes Z_2) \begin{bmatrix} hI_N \otimes A^T \\ -hL \otimes K^T B^T \\ \mathbf{0} \\ -hL \otimes K^T B^T \end{bmatrix}^T \\
&-\begin{bmatrix} \left(I_N - \frac{1}{N}\mathbf{11}^T\right) \otimes Z_2 & \left(I_N - \frac{1}{N}\mathbf{11}^T\right) \otimes \left(S^T - Z_2\right) & -\left(I_N - \frac{1}{N}\mathbf{11}^T\right) \otimes S^T & \mathbf{0} \\ * & \left(I_N - \frac{1}{N}\mathbf{11}^T\right) \otimes \left(2Z_2 - S - S^T\right) & \left(I_N - \frac{1}{N}\mathbf{11}^T\right) \otimes \left(S^T - Z_2\right) & \mathbf{0} \\ * & * & \left(I_N - \frac{1}{N}\mathbf{11}^T\right) \otimes Z_2 & \mathbf{0} \\ * & * & * & \mathbf{0} \end{bmatrix} \\
&= \begin{bmatrix} \Pi_1 & \Pi_2 & \Pi_3 & \Pi_4 \\ * & \Pi_6 & \Pi_7 & \mathbf{0} \\ * & * & \Pi_9 & \mathbf{0} \\ * & * & * & \Pi_{10} \end{bmatrix} + \begin{bmatrix} hI_N \otimes A^T \\ -hL \otimes K^T B^T \\ \mathbf{0} \\ -hL \otimes K^T B^T \end{bmatrix} (I_N \otimes Z_2) \begin{bmatrix} hI_N \otimes A^T \\ -hL \otimes K^T B^T \\ \mathbf{0} \\ -hL \otimes K^T B^T \end{bmatrix}^T \le \mathbf{0}
\end{aligned}
$$

By utilizing the Schur complement, we obtain the above bilinear matrix inequalities (13). Furthermore, considering that

$$
\begin{aligned}
\dot{V}(t) &\le \tilde{\dot{V}}(t) = \chi^T(t)\Omega\chi(t) - \chi^T(t)[e_1(L^2 \otimes Q)e_1^T + e_1(L^2 \otimes K^T RK)e_1^T]\chi(t) \\
&\le -\chi^T(t)[e_1(L^2 \otimes Q)e_1^T + e_1(L^2 \otimes K^T RK)e_1^T]\chi(t) \\
&= -\sum_{i=1}^{N} \left\{ \left[\sum_{j\in N_i}(x_i(t) - x_j(t))\right]^T Q \left[\sum_{j\in N_i}(x_i(t) - x_j(t))\right] + u_i(t)^T R u_i(t) \right\},
\end{aligned}
$$

by integrating both sides of the above inequality from 0 to T, we have

$$
\begin{aligned}
&\sum_{i=1}^{N} \int_0^T \left\{ \left[\sum_{j\in N_i}(x_i(t) - x_j(t))\right]^T Q \left[\sum_{j\in N_i}(x_i(t) - x_j(t))\right] + u_i(t)^T R u_i(t) \right\} dt \\
&\quad \le \int_0^T -\dot{V}(t)\, dt = V(0) - V(T).
\end{aligned}
$$

When $T \to \infty$, we have

$$
\begin{aligned}
&\sum_{i=1}^{N} \int_0^\infty \left\{ \left[\sum_{j\in N_i}(x_i(t) - x_j(t))\right]^T Q \left[\sum_{j\in N_i}(x_i(t) - x_j(t))\right] + u_i(t)^T R u_i(t) \right\} dt \\
&\quad \le V(0) \\
&\quad = \sum_{i=1}^{N} \xi_i^T(0) P \xi_i(0) + \sum_{i=1}^{N} \int_{-h}^{0} \xi_i(s)^T Z_1 \xi_i(s)\, ds + h \sum_{i=1}^{N} \int_{-h}^{0} \int_{\theta}^{0} \dot{\xi}_i(s)^T Z_2 \dot{\xi}_i(s)\, ds\, d\theta \\
&\quad = x^T(0)\left[\left(I_N - \frac{1}{N}\mathbf{11}^T\right) \otimes P\right] x(0) + \int_{-h}^{0} x^T(s)\left[\left(I_N - \frac{1}{N}\mathbf{11}^T\right) \otimes Z_1\right] x(s)\, ds \\
&\qquad + h \int_{-h}^{0} \int_{\theta}^{0} \dot{x}^T(s)\left[\left(I_N - \frac{1}{N}\mathbf{11}^T\right) \otimes Z_2\right] \dot{x}(s)\, ds\, d\theta.
\end{aligned}
$$

This completes the proof. □

Remark 3.1. The consensus conditions for co-design of controller gain matrix K, triggering parameters Φ and ρ can be expressed as matrix inequalities. Next, the event based guaranteed cost consensus problem will be reformulated to a standard continuous constrained optimization problem with linear objective function, linear and bilinear matrix inequalities constraints.

Remark 3.2. The approach used in this study can be extended to multi-agent systems with directed graph and switching topologies if we make some necessary changes of the Laplacian matrix in the bilinear matrix inequalities (13). If the wireless communication is involved, issues of time-delay, packet dropouts and quantization can be addressed with similar methods as those in networked control systems [21–23].

Assume that the initial state of each agent is bounded, i.e., $x_i = Ww_i, w_i^T w_i \le 1, i = 1, \dots, N$, where W is a given real-valued constant matrix. Considering that

$$x^T(0)\left[\left(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\right) \otimes P\right]x(0) \le N\lambda_{\max}\left(W^T P W\right)$$

$$\int_{-h}^{0} x^T(s)\left[\left(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\right) \otimes Z_1\right]x(s)\, ds \le N\lambda_{\max}\left(W^T Z_1 W\right)\lambda_{\max}\left(\int_{-h}^{0} e^{(A^T+A)s}\, ds\right)$$

$$h\int_{-h}^{0}\int_{\theta}^{0} \dot{x}^T(s)\left[\left(I_N - \frac{1}{N}\mathbf{1}\mathbf{1}^T\right) \otimes Z_2\right]\dot{x}(s)\, ds\, d\theta$$
$$\le hN\lambda_{\max}(W^T Z_2 W)\lambda_{\max}\left(\int_{-h}^{0}\int_{\theta}^{0} e^{A^T s}A^T A e^{As}\, ds\, d\theta\right)$$

and defining $c_1 = \lambda_{\max}\left(\int_{-h}^{0} e^{(A^T+A)s}\, ds\right)$, $c_2 = h\lambda_{\max}\left(\int_{-h}^{0}\int_{\theta}^{0} e^{A^T s}A^T A e^{As}\, ds\, d\theta\right)$, then, we can relax the upper bound J^* to

$$J^*(\alpha, \beta, \gamma) = N\alpha + Nc_1\beta + Nc_2\gamma,$$

where

$$W^T P W \le \alpha I, \quad W^T Z_1 W \le \beta I, \quad W^T Z_2 W \le \gamma I. \tag{15}$$

As a result, the associated optimization problem can be formulated as

$$\begin{aligned} &\min J^*(\alpha, \beta, \gamma) = N\alpha + Nc_1\beta + Nc_2\gamma \\ &\quad \text{s.t.} \quad (13) \text{ and } (15) \end{aligned} \tag{16}$$

Remark 3.3. The optimization problem involving linear objective function with linear and bilinear matrix inequalities can be solved by recently developed bilinear matrix inequality (BMI) solvers [6,9]. Note that by using dependent slack variables and fixing one of the coupling variables in BMI, the BMI will be reduced to LMI (linear matrix inequality). However, in the treatment process, the feasible space may become much smaller or even empty, causing the corresponding results to have large conservation or make no sense. In addition, the conservatism of the main results also comes from at least other three aspects: (1) the design of event-triggering condition; (2) the selection of the Lyapunov candidate; (3) the techniques to deal with the time-delay, which will be further considered in our future work.

4. Simulation example

Consider a scenario where four agents are to reach some sort of agreement, with the communication topology given in Fig. 2, which is also used in [4,15].

The adjacency matrix $\mathcal{A}$ and the degree matrix $\mathcal{D}$ are

$$\mathcal{A}=\begin{pmatrix}0&1&1&0\\1&0&1&0\\1&1&0&1\\0&0&1&0\end{pmatrix},\quad \mathcal{D}=\begin{pmatrix}2&0&0&0\\0&2&0&0\\0&0&3&0\\0&0&0&1\end{pmatrix}.$$

Then, the Laplacian matrix is given by

$$L=\mathcal{D}-\mathcal{A}=\begin{pmatrix}2&-1&-1&0\\-1&2&-1&0\\-1&-1&3&-1\\0&0&-1&1\end{pmatrix}.$$

Let A, B, Q, R and W be

$$A=\begin{pmatrix}0.2&1&0\\-0.1&-0.5&0.1\\0.1&-0.1&-0.2\end{pmatrix},\quad B=\begin{pmatrix}1\\0\\1\end{pmatrix},\quad Q=\mathrm{diag}\{1,1,1\},\quad R=0.2,$$

$W=\mathrm{diag}\{1.5,1.5,1.5\}$, the sampling period $h=0.02$ and the triggering parameter $\rho=0.2$.

We can verify that A has a positive eigenvalue 0.0949 and so it is unstable. In addition, the rank of the controllability matrix involving system matrix A and input matrix B is 3 and so it is stabilizable.

In [32], a general purpose BMI solver PENBMI [12], which combines ideas of the (exterior) penalty and (interior) barrier methods with the augmented Lagrangian method, is used to solve the nonlinear semi-definite optimization problem. Solving the optimization problem (16) using the same method yields

$$\Phi=\begin{pmatrix}9.8366&3.8951&4.3475\\3.8951&1.5424&1.7215\\4.3475&1.7215&1.9215\end{pmatrix},\quad K=[2.5613,1.0142,1.1320],$$

and the least upper bound $J^*(\alpha,\beta,\gamma)=836.1529$.

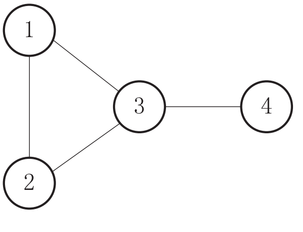
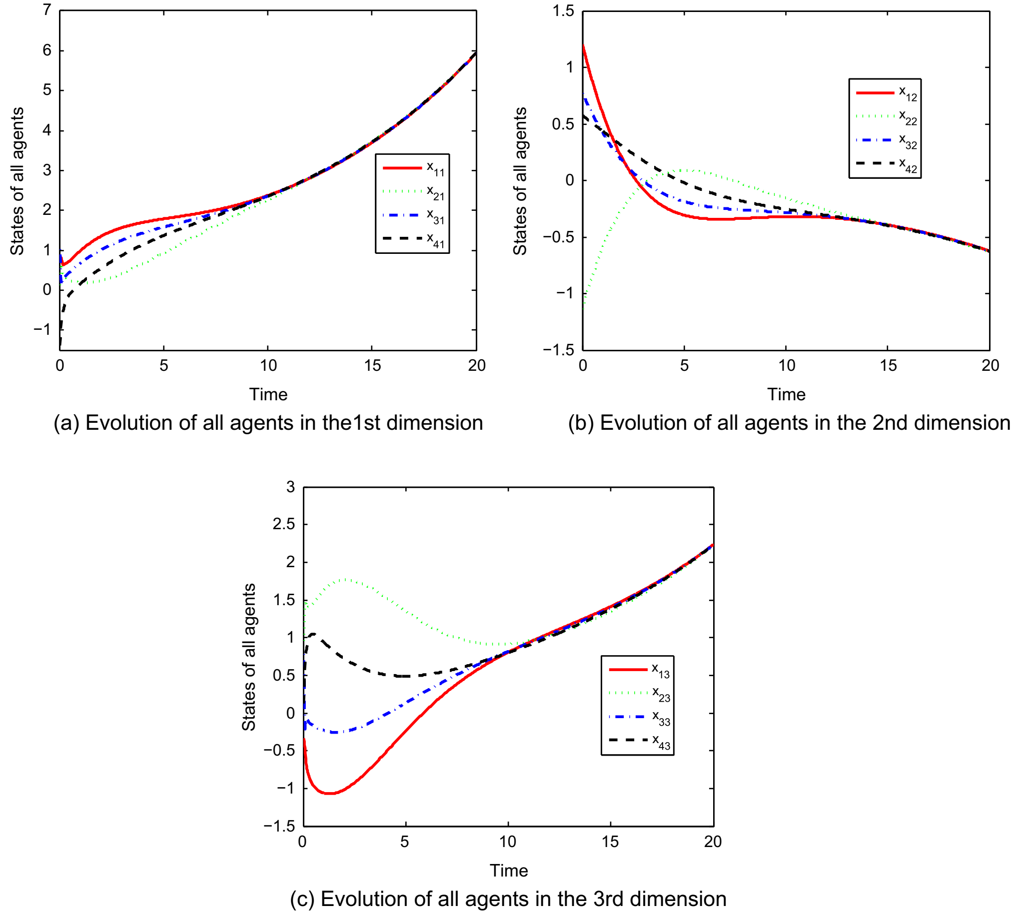


Fig. 2. Communication topology.

Remark 4.1. As can be seen from the bilinear matrix inequalities (13), there exist $3n^2 + mn + 2n$ variables in total; as a result, the time complexity can be estimated as $\mathcal{O}(n^6)$ based on the interior point methods when using PENBMI. For the given instance, $m = 1$, $n = 3$, and the computational time for solving the optimization problem is 0.7572 s in total on Intel(R) Core (TM) i3-2310M CPU at 2.10 GHz under Window 7 environment.

The trajectories of all the agents in each dimension and the trajectories of all the agents in three-dimensional space are illustrated in Figs. 3 and 4, respectively. It is obvious that the consensus is achieved by using the proposed approach.

Then, we study the case when using different values of triggering parameter ρ, the comparisons on whether or not considering the cost functional (11) as objective function regarding energy consumption are given in Table 1. It is shown that significant amount of energy consumption is reduced if the cost functional (11) is involved, especially when the value of the triggering parameter ρ is large. That is to say, incorporating the event based scheme with guaranteed cost performance is necessary for consensus of multi-agent systems with energy constrained embedded microprocessors.



(a) Evolution of all agents in the1st dimension

(b) Evolution of all agents in the 2nd dimension

(c) Evolution of all agents in the 3rd dimension

Fig. 3. Evolution of all agents for each dimension.

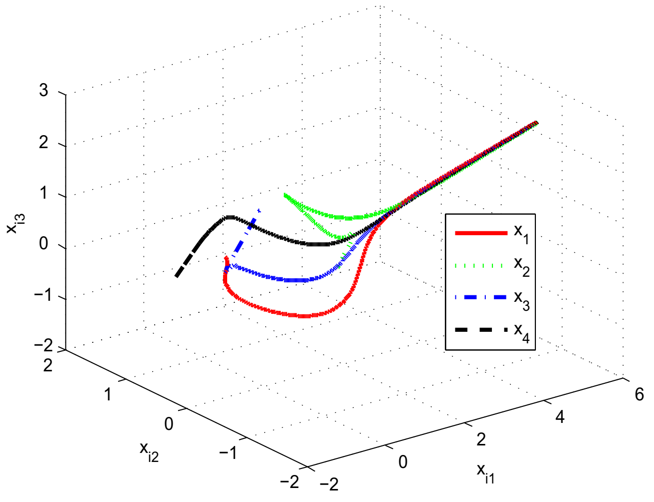


Fig. 4. Evolution of all agents in three dimensional space.

Table 1
Energy consumption with and without guaranteed cost.

J^*	ρ				
	0.05	0.1	0.15	0.2	0.25
Without	3.0606e6	3.0661e6	5.1792e6	4.0013e7	2.8958e8
With	831.2327	832.4816	833.9547	836.1529	838.1478

5. Conclusion and future work

Taking the energy consumption into consideration, the event based guaranteed cost consensus for distributed multi-agent systems with general LTI dynamics was studied in this paper. To reach a consensus, the design of the controller gain matrix and the triggering parameters with guaranteed cost performance were formulated as a continuous constrained optimization problem expressed as a linear objective function with linear and bilinear matrix inequalities constraints. Numerical results validated the effectiveness of the proposed approach and also showed the necessity of involving the guaranteed cost performance with the event based control scheme. In our future work, we will extend the proposed approach to multi-agent systems with uncertainty, directed graph and switching topologies.

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